

Figure 2. Performance evaluation flow of routing protocols in wireless networks.

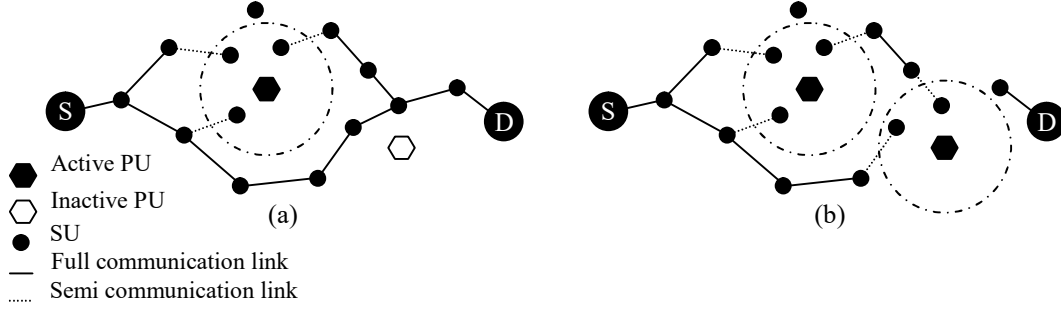


Figure 3. Network model and communication links in CRAHNs.

work in this paper is to help network designers select optimal network topology related parameters so that data transmission failures relating to network topology can be eliminated, saving network evaluation effort for network designers.

The rest of this paper is organized as follows. In Section 3, based on geographical location aware greedy routing [21], we propose algorithms to evaluate the connectivity of multi-hop path between arbitrary source node and destination node from network topology perspective with random node distribution. Section 4 shows numerical results of the connectivity of multi-hop path in CRAHNs with various simulation scenarios. Finally, conclusions and future works are given in Section 5.

3. Evaluation of the connectivity of multi-hop path between two arbitrary nodes in CRAHNs and AHNs

3.1. Network model

We consider CRAHNs where N_s secondary users (SUs) coexist with N_p primary users (PUs). All SUs and PUs are uniformly distributed in network area a^2 . Each SU can utilize the licensed frequencies of PUs. We suppose that all PUs are characterized by an on-off transmission following Poisson distribution with average active rate λ_{PU} . All senders transmit packets with constant power P_t and receivers can receive packets successfully if packet reception power is higher than a threshold P_{th} . The effective transmission range of wireless node is

determined by P_t , P_{th} , and its operating frequency. The transmission area and forwarding area of each node are considered as circular area and semicircular area, respectively. The same network model is used for AHNs except that there are no PUs in the networks.

An illustrative example of network model and available communication links in CRAHNs is presented in Figure 3. Semi communication link is the wireless link between two nodes which are in the transmission range of each other. However, the sender is out of transmission range of active PU and the receiver is in the transmission range of active PU. Obviously, the multi-hop path from source node to destination must consist of all full communication links because all intermediate SUs on the multi-hop path in CRAHNs act as receiver and sender in order to forward data packets. In Figure 3(a), when only one PU is active, a multi-hop path can be established between source SU and destination SU. In contrast, when two PUs are active as in Figure 3(b), no multi-hop paths between these two nodes exist. Details of the method for evaluating the connectivity of multi-hop path in CRAHNs are presented below.

3.2. Our proposed algorithm

In this section, we present our proposed algorithm to evaluate the connectivity of multi-hop path from an arbitrary source node to an arbitrary destination node in AHNs and CRAHNs. With initial input network parameters such as random node location, operating frequency of wireless nodes, active state of each PU, criteria of forwarder selection (i.e. geographical location aware greedy routing [21] used in this paper), our

proposed algorithm can check all possible paths between two arbitrary nodes considered as source node and destination node and return whether these two nodes are connected, i.e. there is at least one communication path between them. It should be noted that with the same initial input network parameters, if we apply Dijkstra's algorithm [22] to such wireless networks, the algorithm may not work because it requires connected graph as an input. However, in AHNs and CRAHNs, due to limited transmission range, random location of wireless node (SU), and random active state of PU, there may exist no path between two arbitrary nodes. Thus, the information of connected network graph in wireless networks such as AHNs and CRAHNs is not available initially. Another issue is Dijkstra's algorithm does not have the mechanism for determining forwarders based on routing algorithm as in our proposed path finding algorithm.

The pseudocode of our proposed algorithm used in conventional AHNs is presented in Figure 4.

The explanations for the algorithm to evaluate connectivity in AHNs with given wireless node locations are as follows:

Lines 1-2: If the destination node is within the transmission range of source node, source node can communicate directly to destination nodes without needing the support of any intermediate nodes, or the connectivity is 1. The process of finding paths is ended.

Lines 4-6: If source node does not have any neighboring nodes as forwarders, no paths from source node to destination node can be established, or the connectivity is 0. The process of finding path is also ended.

```

1  If ( $d(src, dest) \leq R$ )
2    connectivity = 1; // dest in the radio range of src → connectivity = 1
3  Else
4    fwdnode = fwdnode_chk(src)
5    If (fwdnode = {})
6      connectivity = 0; // src does not have any forwarders → multi-hop path cannot be established
7    Else
8      gonna_chk_node = gonna_chk_node + {fwdnode}; // add fwdnode to check list
9      hop_count(gonna_chk_node) = 1; // initialize hop count
10     While (gonna_chk_node ≠ {})
11       chk_node = gonna_chk_node;
12       For each chk_node
13         hc_chk_node = hop_count(chk_node); // get hop count to current node
14         fwdnode = fwdnode_chk(hc_chk_node); // find forwarders of current node
15         For each node ∈ fwdnode
16           If (node ∉ already_chk_node)
17             already_chk_node = already_chk_node + {node}; // add fwd to already checked list
18             hop_count = hop_count + {hop_count(node)}; // update hop count table
19             gonna_chk_node = gonna_chk_node + {node}; // add forwarder to gonna check list
20           Else
21             If (hc_chk_node + 1 < hop_count(node))
22               hop_count(node) = hop_count(node) + 1; // increase hop count to forwarder
23               gonna_chk_node = gonna_chk_node + {node}; // add fwd to gonna check list
24           End-For
25           gonna_chk_node = gonna_chk_node \ {chk_node}; // remove checked node from the check list
26           If (dest ∈ already_chk_node)
27             connectivity = 1; // if dest is reached, multi-hop path can be established
28           End-For
29         End-While
30       If (dest ∉ already_chk_node)
31         connectivity = 0; // multi-hop path cannot be established

```

Figure 4. The pseudocode of our proposed algorithm to analyze the connectivity of multi-hop path between two arbitrary nodes in AHNs with given random wireless node locations.

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1  For each active  $PU \in PU$ 
2      If ( $d(src, active\ PU) \leq R \parallel d(dest, active\ PU) \leq R$ )
3          connectivity = 0; stop = 1; break; // src or dest is affected by active PU → no routing path
4  End-For

5  If ( $d(src, dest) \leq R \ \&\& \ stop \neq 1$ )
6      hop count = 1; // dest in the radio range of src → hop count = 1
7  If ( $d(src, dest) > R \ \&\& \ stop \neq 1$ )
8      fwdnode = fwdnode_chk(src)
9      If (fwdnode = {})
10         connectivity = 0; // src does not have any forwarders → multi-hop path cannot be established
11     Else
12         For each node  $\in fwdnode$ 
13             If ( $d(node, active\ PU) \leq R$ )
14                 fwdnode = fwdnode \ {node}; // not consider forwarders affected by active PU
15         End-For
16         gonna_chk_node = gonna_chk_node + {fwdnode}; // add fwdnode to gonna check list
17         hop_count(gonna_chk_node) = 1; // initialize hop count
18         While (gonna_chk_node  $\neq \{\}$ )
19             chk_node = gonna_chk_node;
20             For each chk_node
21                 hc_chk_node = hop_count(chk_node); // get hop count to current node
22                 fwdnode = fwdnode_chk(hc_chk_node); // find forwarders of current node
23                 For each node  $\in fwdnode$ 
24                     If ( $d(node, active\ PU) \leq R$ )
25                         fwdnode = fwdnode \ {node}; // not consider forwarders affected by active PU
26                 End-For
27                 For each node  $\in fwdnode$ 
28                     If (node  $\notin$  already_chk_node)
29                         already_chk_node = already_chk_node + {node}; // add fwd to already checked list
30                         hop_count = hop_count + {hop_count(node)}; // update hop count table
31                         gonna_chk_node = gonna_chk_node + {node}; // add forwarder to gonna check list
32                     Else
33                         If (hc_chk_node + 1 < hop_count(node))
34                             hop_count(node) = hop_count(node) + 1; // increase hop count to forwarder
35                             gonna_chk_node = gonna_chk_node + {node}; // add fwd to gonna check list
36                     End-For
37                 gonna_chk_node = gonna_chk_node \ {chk_node}; // remove checked node from the check list
38                 If (dest  $\in$  already_chk_node)
39                     hop count = 1; // if dest is reached, multi-hop path can be established
40             End-For
41         End-While
42     If (dest  $\notin$  already_chk_node)
43         connectivity = 0; // multi-hop path cannot be established

```

Figure 5. The pseudocode of our proposed algorithm to analyze the connectivity of multi-hop path between two arbitrary nodes in CRAHNs with given random wireless node locations.

Lines 8-29: Since our proposed algorithm aims to find all possible paths from source node to destination node with given random node location, operating frequency of wireless nodes and return the connectivity of multi-hop path between these two nodes, the process of forward path searching from source node are follows: For each currently checked node, find its forwarders. These forwarders are added to the list of going-to-be checked nodes if (i) they have not been checked or (ii) the hop count from source to them is less than previous values. Node which already checked for its forwarders is removed from the list of going-to-be-checked nodes. This process is repeated for the next node in the list of going-to-be checked nodes until this list is empty.

Lines 30-31: If the list of going-to-be-checked nodes is empty but still the destination node cannot be reached, it means that no paths from source node to destination node can be established, or the connectivity is 0.

The pseudocode of our proposed algorithm used in CRAHNS is presented in Figure 5. Since PUs exist in CRAHNS, the algorithm used to evaluate the connectivity of multi-hop path in CRAHNS shown in Figure 5 has several different parts compared with the algorithm used to evaluate the connectivity of multi-hop path in conventional AHNs shown in Figure 4. Specifically, **Lines 1-4:** The algorithm returns the connectivity is 0 (multi-hop path cannot be established) when either source node or destination node is influenced by active PU(s). **Lines 12-15** and **Lines 23-26:** The algorithm eliminates wireless nodes affected by active PU(s) from the list of potential forwarders because they are considered as not be able to participate in forming multi-hop path from source node to destination node. It is because the concepts of forwarders in conventional AHNs and CRAHNS are different. More specifically,

- *In conventional AHNs:* forwarders of a specific node are the nodes in its forwarding area.
- *In CRAHNS:* forwarders of a specific node are the node in its forwarding area and not affected by the presence of any active PUs.

4. Simulation analysis of the connectivity of multi-hop path between two arbitrary nodes in CRAHNS and AHNs

4.1. Simulation environment and method

In this section, we present statistical simulation method to obtain simulation results of network connectivity in CRAHNS by using our proposed algorithms. We evaluate the connectivity in CRAHNS with different settings of network parameters such as the number of SUs and PUs, network size, network operating frequency, and the average active rates of PU. All wireless nodes have transmission power $P_t = 10^{-3}$ W and reception power threshold $P_{th} = 1.58 \times 10^{-12}$ W. Those network parameters are used for all evaluating scenarios if not further mentioned.

MATLAB simulation tool is used to obtain simulation results for investigating the connectivity of multi-hop path in CRAHNS. The simulation is conducted on a computer workstation equipped with 3.5 GHz (Intel Core i5 – 3570 Quad) processor, 4 GB of RAM and Windows 7. The simulation time for each value of simulation results varies from 1.4 hours to 4.9 hours, depending on the values of input network parameters such as number of wireless nodes, operating frequency of network, average active rate of PU. Figure 6 illustrates four network topologies among numerous ones randomly created during simulation with 5 PUs and 100 SUs in a network area of $1000 \text{ m} \times 1000 \text{ m}$. There is one licensed frequency layer with $f = 2.4$ GHz.

The detailed simulation processes are as follows.

- **Step 1:** We create a square network area with size of $a \times a$. Next, for each frequency layer, we place N_{SU} secondary users and N_{PU} primary users using uniform distribution into the network area. The number of activating times in a unit of time of each PU follows Poisson distribution with average rate λ_{PU} .
- **Step 2:** One operating frequencies in the networks (i.e. licensed frequencies from PUs perspective, unlicensed frequencies from SUs perspective) are selected. The transmission ranges of wireless nodes, i.e. for both PUs and SUs, are determined by their transmission power, reception power threshold, and operating frequencies.
- **Step 3:** Two arbitrary SU nodes in the network are selected as source node and destination node. For each SU, a forwarding direction from it to destination node is use to determine the forwarding area, i.e. semicircular area, of that SU. If that SU has at least one neighboring SU in its forwarding region while there is no active PU in its transmission area, it is said that there is at least one communication link for that SU. The same procedure is applied for next SUs on the multi-hop path.

We repeat the above processes Ω times for each operating frequency. For each time, a new distribution of wireless nodes is created and new arbitrary source SU and destination SU are selected. We perform the simulation of each evaluating scenario with huge number of experiments, that is, $\Omega = 50,000$. Obviously, the simulation result of network connectivity of multi-hop path is calculated as

$$\frac{\#connected_paths}{\Omega} \xrightarrow{\Omega \rightarrow \infty} P_{con}^{SU} \quad (1)$$

Figure 6 illustrates 4 over 50,000 random network topologies with uniform node distribution. In these random network topologies, red node is an arbitrary source SU, green node is an arbitrary destination SU, black nodes are active PUs, red dotted lines refer to the forwarding directions from current SUs to destination SU, and blue lines and blue dotted lines present full

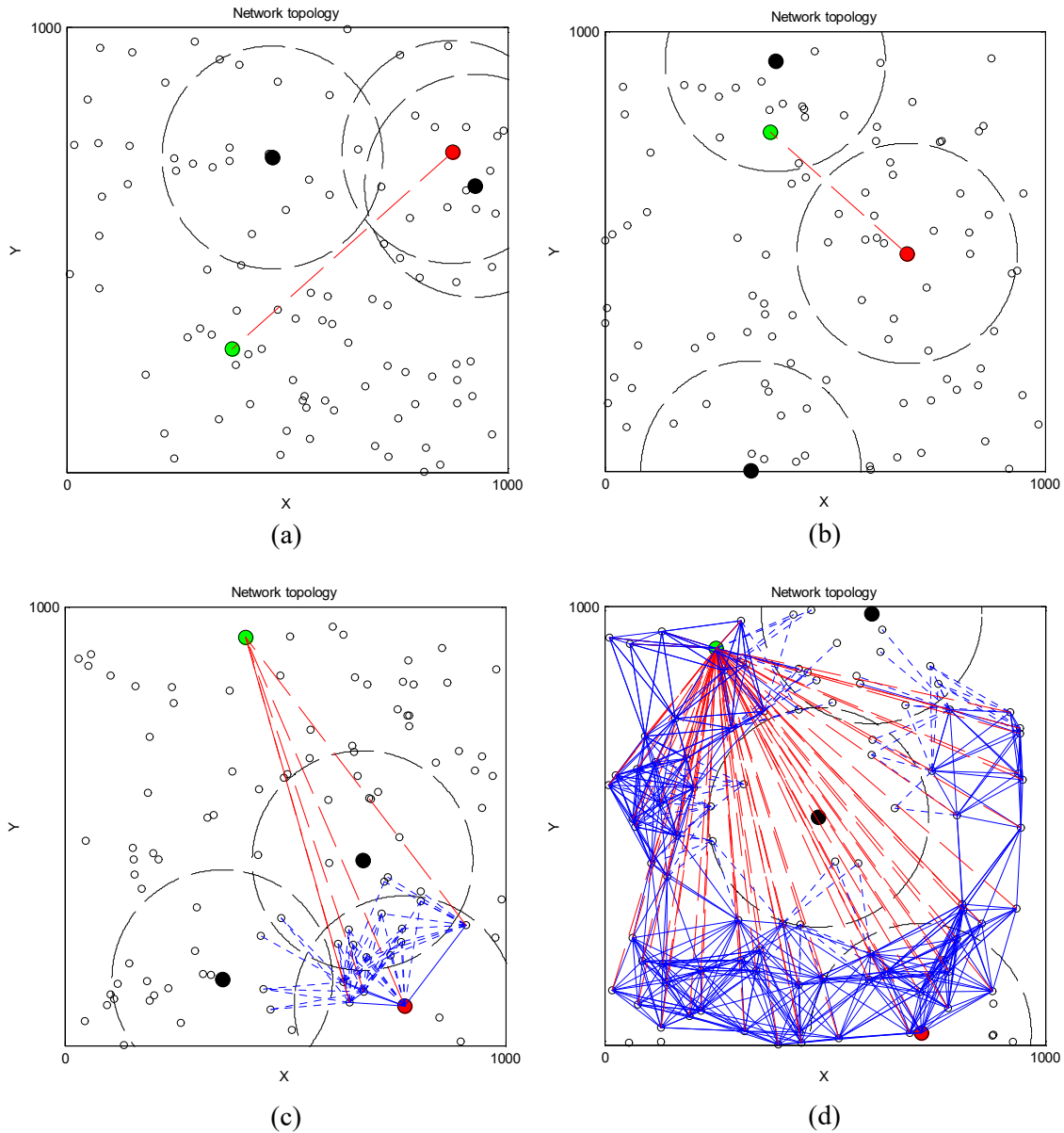


Figure 6. Illustrations of 4 over 50000 random network topologies used in simulation to evaluate the connectivity of multi-hop path between two arbitrary nodes in CRAHNs.

communication links and semi communication links between two SUs, respectively. As shown in Figure 6, the multi-hop path from source SU to destination SU may not be successfully established because of the following reasons: i) source SU is influenced by active PUs as in Figure 6(a), ii) destination SU is influenced by active PUs as in Figure 6(b), or iii) intermediate SUs are influenced by active PUs as in Figure 6(c). It should be noted that establishing multi-hop paths in CRAHNs may not be successful when a SU does not have any neighboring nodes in its forwarding area, as also happens in AHNs. Figure 6(d) is the case that multi-hop path is successfully established. From Figure 6(d), we can see that multi-hop path has to detour to avoid interfering with active PUs.

4.2. Numerical results and discussions

Scenario 1: The impact of node's operating frequency on the connectivity of multi-hop path in CRAHNs and AHNs

In this scenario, we compare the point-to-point link connectivity of conventional AHNs and CRAHNs in the same network conditions. The network settings are as follows. For CRAHNs, there are 5 PUs uniformly distributed in the network area. The average activating rate of PU is $\lambda_{PU} = 0.1$. There is only one frequency layer in the network, i.e. all PUs operate with one license frequency band and all SUs utilize that licensed frequency band to communicate with each other. The number of SUs

in the network is 100. All SUs are also uniformly distributed in network area. For conventional AHNs, network settings are the same as CRAHNs except that there is no PU in the network.

Figure 7 presents the connectivity of multi-hop path in CRAHNs as a function of wireless node's operating frequency compared with conventional AHNs. From Figure 7, we can see some interesting features: i) the connectivity of CRAHNs is always lower than that of conventional AHN. It is due to the presence of PUs in CRAHNs, making multi-hop paths more difficult to be established. Moreover, when the operating frequency of wireless node (SU) is low (corresponding to large effective transmission range), the difference between connectivity of multi-hop path in CRAHNs and AHNs is more significant when the operating frequency of wireless nodes is high (corresponding to small effective transmission range). It is because in CRAHNs, when effective transmission range of wireless node is large, the possibility of having active PUs inside the effective transmission range of SUs is higher, leading to lower connectivity of multi-hop path. In contrast, in AHNs, there is no influence of PU's presence on the connectivity of multi-hop path. ii) the connectivity of AHNs does not have parabolic pattern as that of CRAHNs. The reason behind this feature is there is no PU in AHNs. Whereas in CRAHN, when operating frequency of SU is low (corresponding to large effective transmission range), the possibility that an SU is influenced by active PUs strongly dominates the probability that that SU has neighboring nodes in forwarding area, resulting in low connectivity of multi-hop path. As operating frequency of SU is higher (corresponding to shorter effective transmission range), the influence of active PU's presence is relieved, leading to higher connectivity of multi-hop path. iii) when operating frequency of wireless node continues to increase (corresponding to shorter effective transmission range), the connectivity of multi-hop path in both AHNs and CRAHNs remarkably decreases and almost similar at high operating frequency of wireless node. It happens because shorter effective transmission range reduces the possibility that SU has neighboring nodes in its forwarding area and increases the hop count of multi-hop path (which also means increasing the difficulty of path establishment) at the same time.

Scenario 2: The impact of the number of SUs on the connectivity of multi-hop path in CRAHNs and AHNs

In CRAHNs, a SU communicates with other SUs in the way that their communication does not interfere with the operations of PUs. Therefore, network connectivity in CRAHNs implies the connectivity among SUs in the network. In this scenario, we investigate how the number of SUs influences the network connectivity of CRAHNs provided that network size and the number of PUs are fixed. We also compare the connectivity of CRAHN with conventional AHNs when two networks have the same node (SU) density. The number of PUs in the network is 5. The average active rate $\lambda_{PU} = 0.1$. There is only one

operating frequency (i.e. one unlicensed frequency band from SU's perspective) in the network, which is set at 2.4 GHz.

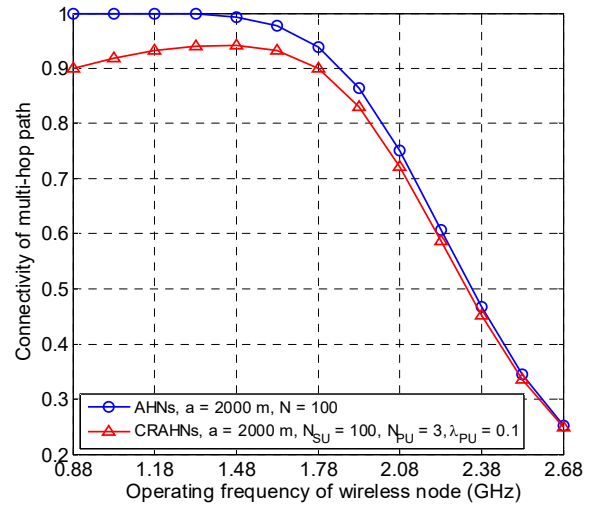


Figure 7. The connectivity of multi-hop path in CRAHNs and conventional AHNs with different operating frequencies; network size $a = 2000$ m; number of SUs = 100, number of PUs = 3, average active rate of PU $\lambda_{PU} = 0.1$.

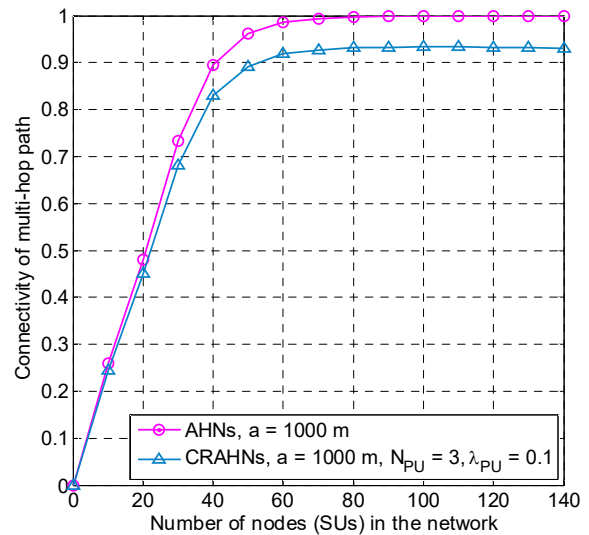


Figure 8. The connectivity of CRAHNs compared with conventional AHNs with different numbers of wireless nodes (SUs); the number of PUs = 3, average active rate of PU $\lambda_{PU} = 0.1$, one frequency layer with operating frequency $f = 2.4$ GHz.

Figure 8 shows the connectivity of CRAHNs compared with conventional AHNs as function of the number of wireless nodes (SUs). From Figure 8, the following distinguishing features can be observed: i) the connectivity of multi-hop path in both AHNs and CRAHNs rapidly increases as the number of wireless

nodes (SUs) increases. However, the connectivity of CRAHNs is always lower than that of AHNs due to the influence of active PUs. ii) the connectivity of multi-hop path in both AHNs and CRAHNs reach *steady state* when the number of wireless nodes (SUs) is higher than a certain value (i.e. 80 nodes in this simulation scenario). In steady state, the connectivity of multi-hop path is almost constant when the number of wireless nodes further increases. From this observation, we can determine the optimal number of wireless nodes which provide the highest connectivity of multi-hop path while saving network resources such as energy consumption, bandwidth usage in the networks. iii) the connectivity of multi-hop path in CRAHNs at steady state is lower than that of AHNs because active PUs occupied certain network area, thus reduces the possibility of successfully establishing routing path of SUs.

Scenario 3: The impact of node's operating frequency on the connectivity of multi-hop path in CRAHNs and AHNs

In this scenario, we study how PU's node density affects network connectivity in CRAHNs by varying number of PUs from 1 to 7 while keeping number of SUs at 100. Average PU's activating rate $\lambda_{PU} = 0.1$. Network size is $2000 \text{ m} \times 2000 \text{ m}$.

Figure 9 shows the connectivity of multi-hop path in CRAHNs with different number of PUs. For the purpose of evaluating the degree of influence of PU density on the connectivity of multi-hop path on different frequency bands, we plot the connectivity of multi-hop path as a function of operating frequency of wireless node and with different numbers of PUs in the networks. We can see the following important features: i) the increase in the number of PUs has stronger influence on the connectivity of multi-hop path when SU operate on low frequency bands (corresponding to large effective transmission range) compared with that when SU operate on high frequency bands (corresponding to short effective transmission range). It is because the average number of PUs in the transmission range of a SU is proportional to transmission area which depends on operating frequency of SU, ii) the parabolic relationship between connectivity of multi-hop path and the operating frequency of wireless node is significantly relaxed as the number of PUs in the networks decreases.

Scenario 4: The impact of network size on the connectivity of multi-hop path in CRAHNs

In this scenario, we study how network size affects network connectivity in CRAHNs. We use different values of network size, i.e. $1000 \text{ m} \times 1000 \text{ m}$, $2000 \text{ m} \times 2000 \text{ m}$, and $3000 \text{ m} \times 3000 \text{ m}$ to create vast change of SU's density, PU's density, and the average distance between arbitrary source node and destination node. The number of PUs is 3. The average active rate of PU is 0.1. It should be noted that changing in network size results in changing of SU's density, PU's density, and the average distance between arbitrary source node and destination

node at the same time. Thus, variation in network size may have the strongest effects on the connectivity of multi-hop path in CRAHNs compared with those in previous scenarios. To provide insights of the impact of network size, we plot the connectivity of multi-hop path in CRAHNs as functions of operating frequency of wireless node and the number of SUs in the network with different network size as shown in Figure 9 and Figure 10, respectively. We also compared the connectivity of multi-hop path in CRAHNs with that in AHNs.

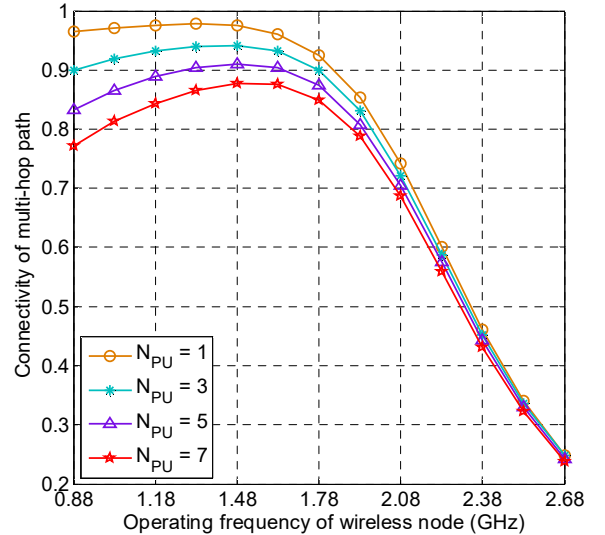


Figure 9. The connectivity of multi-hop path in CRAHNs with different number of PUs; network size = $2000 \text{ m} \times 2000 \text{ m}$, number of SUs = 100, average active rate of PU $\lambda_{PU} = 0.1$, one frequency layer with operating frequency = $0.88 \text{ GHz} \sim 2.68 \text{ GHz}$.

As confirmed in Figure 10, changing in network size remarkably changes the pattern of the connectivity of multi-hop path. Specifically, when network size is small, i.e. $a = 1000 \text{ m}$, the ratio of node's effective transmission area over network size is small. Thus, the length of multi-hop path in terms of hop count is short, and the possibility of finding neighboring node is high, resulting in almost perfect connectivity in AHNs. However, it may not true in CRAHNs, that is, the connectivity of multi-hop path is much lower and increases as operating frequency increase (corresponding to decrease in effective transmission range) because the influence of active PU's is mitigated. Another interesting observation is the difference between the connectivity of multi-hop path in CRAHNs and AHNs is reduced as network size increases. In addition, the connectivity of multi-hop path sharply reduces when network size $a = 2000 \text{ m}$ and $a = 3000 \text{ m}$. It is because the ratio of node's effective transmission area over network area decreases and the average distance between arbitrary source node and destination node rapidly increases, leading to significant increase of the path length in terms of hop count.

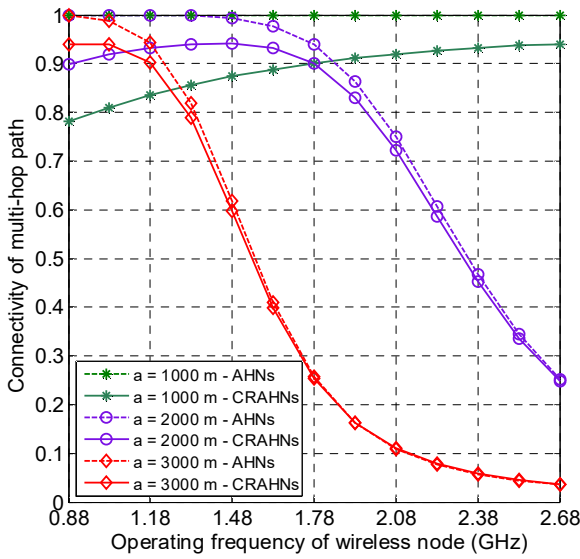


Figure 10. The connectivity of multi-hop path in CRAHNs with different network sizes; number of PUs = 3, number of SUs = 100, average active rate of PU = 0.1, one frequency layer with operating frequency = 0.88 GHz ~ 2.68 GHz.

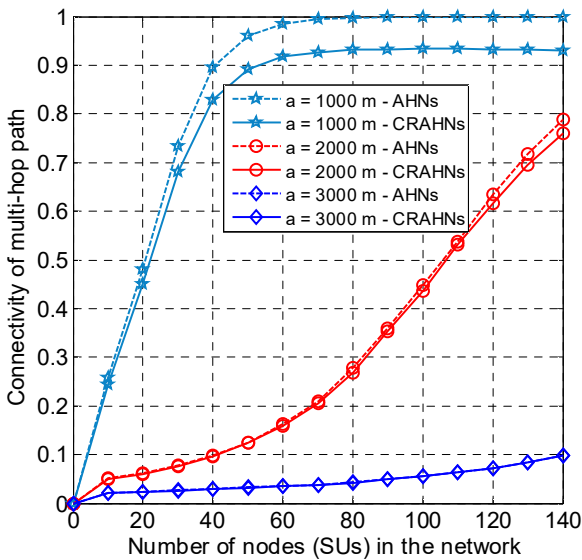


Figure 11. The connectivity of multi-hop path in CRAHNs with different network sizes; number of PUs = 3, average active rate of PUs = 0.1, one frequency layer with operating frequency = 2.4 GHz.

The pattern of the connectivity of multi-hop path in Figure 11 also noticeably changes with different network sizes. When network size $a = 1000$ m, the connectivity of multi-hop path in CRAHNs and AHNs reaches *saturation state* as the number of nodes (SUs) exceeds certain value (80 nodes in Figure 11). In contrast, the connectivity of multi-hop paths in both CRAHNs and AHNs cannot reach saturation state when network size $a = 2000$ m and $a = 3000$ m. It is due to the fact that failures in establishing multi-hop path usually occur because path length in terms

of hop count greatly increases. Especially, when network size $a = 3000$ m, the connectivity of multi-hop path in CRAHNs and AHNs is very low and almost similar.

Scenario 5: The impact of average active rate of PUs on the connectivity of multi-hop path in CRAHNs

In this scenario, we evaluate how the average active rate of PU influences the connectivity of multi-hop path in CRAHNs. There are 100 SUs and 3 PUs in network size of $2000 \text{ m} \times 2000 \text{ m}$. All nodes share one licensed frequency bands with frequency range from 0.88 GHz to 2.68 GHz. The average active rate of PU varies from low rate, i.e. $\lambda_{PU} = 0.1$, to high rate, i. e. $\lambda_{PU} = 0.7$.

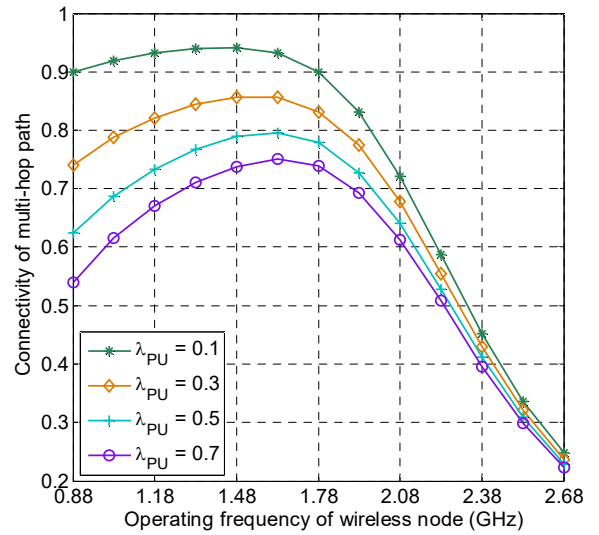


Figure 12. The connectivity of multi-hop path in CRAHNs with different average active rates of PU; network size = $2000 \text{ m} \times 2000 \text{ m}$, number of SUs = 100, number of PUs = 3, one frequency layer with operating frequency = 0.88 GHz ~ 2.68 GHz.

As shown in Figure 12, the average active rate of PU has the same effect on the connectivity of multi-hop path in CRAHNs as that of the number of PUs in Figure 9. Specifically, the connectivity of multi-hop path in CRAHNs is inversely proportional to the increase of average PU's active rate. Moreover, the parabolic pattern of connectivity of multi-hop path in CRAHNs corresponding to operating frequency of wireless node is relieved as the effect of PU presence reduces, i.e. the average active rate of PU decreases.

5. Conclusions and future work

In this paper, we intensively investigate the connectivity of multi-hop path in CRAHNs by using our proposed algorithm and statistical simulation method with different network parameters such as the number of SUs and PUs, operating frequency of the network, network size, and the average active rate of PU. Unlike previous works on connectivity of in the literature, in this paper, we study the

connectivity of multi-hop path between two arbitrary nodes with taking the forwarder selection criteria of routing protocol into consideration, which may lead to major difference in the connectivity of multi-hop path. The simulation results obtained through various experiment scenarios with huge number of random network topologies reveal many interesting and distinguishing feature of the connectivity of multi-hop path in CRAHNs compared with that in AHNs. The results in this paper can be useful guidelines for secondary network planning, for data routing purposes and network performance to provide a reliable communication while saving network resources. Mathematical analysis of the connectivity of multi-hop paths between two arbitrary nodes in different fading channels is considered as our future work.

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