precoders will outperform their BD-LR-ZF and BD counterpart regarding the SEPs, as confirmed by the simulation results in the below section.

3.3. Computational Complexity Analysis

In this sub section, we evaluate the computational complexity of the proposed precoders and compare them with those of LC-RBD-LR-ZF algorithm in [11] and of BD algorithm in [4]. The complexities are evaluated by counting the necessary floating point operations (flops). We assume that each real operation (such as an addition, a multiplication or a division) is counted as a flop. Hence, a complex multiplication and a division require 6 flops and 11 flops, respectively. According to [20], SVD operation of an $m \times n$ complex matrix with $m < n$ requires $4n^2m + 8nm^2 + 9m^3$ flops.

Based on the above assumptions, the computational complexities of the proposed BD-LR-ZF and BD-LR-MMSE precoders are given by:

$$F = F_1 + F_2 + F_3 + F_4 + F_5 \ \ (flops) \tag{28}$$

where F_1 is the number of flops required for SVD operation of the $\tilde{\mathbf{H}}_l$ matrix; F_2 is the number of flops of the multiplication two matrices $\mathbf{H}_l$ and $\mathbf{W}_{BD}^l$ ($l = 1, 2$); F_3 is the number of flops to create $\bar{\mathbf{H}}_l^{LR}$ by the ELR-SLB algorithm in [14]; F_4 is the number of flops to create the matrix $\mathbf{W}_{ZF}^l$ or $\mathbf{W}_{MMSE}^l$ in (11) and (12), respectively; and F_5 is the number of flops for the multiplication two matrices $\mathbf{W}_{BD}$ and $\mathbf{W}_{LP}$.

The number of flops for SVD operations is given by:

$$F_1 = 2(4N_T^2\alpha + 8N_T\alpha^2 + 9\alpha^3) \ (flops) \tag{29}$$

F_2 is calculated to be:

$$F_2 = 2(8N_T\alpha^2 - 2\alpha^2) \ (flops) \tag{30}$$

Since ELR-SLB algorithm is adopted, F_3 is given by:

$$F_3 = 2(24\alpha^3 - 4\alpha^2 + F_{SLB}) \ \ (flops) \tag{31}$$

herein F_{SLB} is the number of flops for the update operation of ELR-SLB algorithm [14], which can only be obtained by using the computer simulation. Note that each update operation in ELR-SLB algorithm requires $(16\alpha + 8)$ flops. The computations of $\lambda_{i,k}$ and $\Delta_{i,k}$ of ELR-SLB algorithm in [16] need 4 flops and 10 flops, respectively. Therefore, F_{SLB} is calculated as follows:

$$\begin{aligned} F_{SLB} &= CUpdate \times (16\alpha + 8) \\ &\quad + CLamda \times 4 + CDelta \times 10 \ \ (flops), \end{aligned} \tag{32}$$

where $CLamda$ is the number of updates $\lambda_{i,k}$, $CDelta$ is the number of updates $\Delta_{i,k}$, $CUpdate$ is the number of updates $t_k^{'}$ and $\tilde{c}^k$ from Steps 7 to Step 9 of ELR-SLB algorithm in [14].

The computational complexity of ZF algorithm is the number of flops to calculate: $(\bar{\mathbf{H}}_1^{LR})(\bar{\mathbf{H}}_1^{LR})^H$, $[(\bar{\mathbf{H}}_1^{LR})(\bar{\mathbf{H}}_1^{LR})^H]^{-1}$ and $(\bar{\mathbf{H}}_1^{LR})^H[(\bar{\mathbf{H}}_1^{LR})(\bar{\mathbf{H}}_1^{LR})^H]^{-1}$ in (11). Therefore, F_4 is calculated to be:

$$F_4 = 2(24\alpha^3 - 4\alpha^2) \ (flops) \tag{33}$$

The computational complexity of the MMSE algorithm is the number of flops to calculate: $\sigma^2\mathbf{U}_1\mathbf{U}_1^H$, $(\bar{\mathbf{H}}_1^{LR})(\bar{\mathbf{H}}_1^{LR})^H + \sigma^2\mathbf{U}_1\mathbf{U}_1^H$, $[(\bar{\mathbf{H}}_1^{LR})(\bar{\mathbf{H}}_1^{LR})^H + \sigma^2\mathbf{U}_1\mathbf{U}_1^H]^{-1}$ and $(\bar{\mathbf{H}}_1^{LR})^H[(\bar{\mathbf{H}}_1^{LR})(\bar{\mathbf{H}}_1^{LR})^H + \sigma^2\mathbf{U}_1\mathbf{U}_1^H]^{-1}$ in [12]. Therefore, in this case, F_4 can be obtained as follows:

$$F_4 = 2(24\alpha^3 - 3\alpha^2 + \alpha + 1) \ (flops) \tag{34}$$

F_5 is given by:

$$F_5 = 8N_T N_R^2 - 2N_R^2 \ \ (flops) \tag{35}$$

From the above analysis results, the total number of flops for the proposed BD-LR-ZF and BD-LR-MMSE precoders are given in (36) and (37), respectively.

$$\begin{aligned} F_{BD-LR-ZF} &= F_1 + F_2 + F_3 + F_4 + F_5 \\ &= 2(4N_T^2\alpha + 8N_T\alpha^2 + 9\alpha^3) + 2(8N_T\alpha^2 - 2\alpha^2) \\ &\quad + 2[24\alpha^3 - 4\alpha^2 + CUpdate \times (16\alpha + 8) \\ &\quad + CLamda \times 4 + CDelta \times 10] + 2(24\alpha^3 - 4\alpha^2) \\ &\quad + 8N_T N_R^2 - 2N_R^2 \ \ (flops) \end{aligned} \tag{36}$$

$$\begin{aligned} F_{BD-LR-MMSE} &= F_1 + F_2 + F_3 + F_4 + F_5 \\ &= 2(4N_T^2\alpha + 8N_T\alpha^2 + 9\alpha^3) + 2(8N_T\alpha^2 - 2\alpha^2) \\ &\quad + 2[24\alpha^3 - 4\alpha^2 + CUpdate \times (16\alpha + 8) \\ &\quad + CLamda \times 4 + CDelta \times 10] + 2(24\alpha^3 - 3\alpha^2 + \alpha + 1) \\ &\quad 8N_T N_R^2 - 2N_R^2 \ \ (flops) \end{aligned} \tag{37}$$

The complexities all of the precoders under consideration are summarized in Table I.

Precoding algorithms	Complexity (flops)	Complexity level
LC-RBD-LR-ZF	$K[6(N_R-N_u)(N_R+N_T-N_u)^2+4(N_R-N_u)(N_R+N_T-N_u)-(N_R+N_T-N_u)^2$ $-(N_R+N_T-N_u)]+K(8N_T^2N_u-2N_TN_u)+K(16N_u^2N_T-2N_uN_T+8N_u^3-2N_u^2+F_{LLL})$ $+K(8N_u^3+16N_u^2N_T-2N_u^2-2N_uN_T)+8KN_T^2N_R-2N_TN_R$	$O(KN_T^2N_R)$
BD	$K[4N_T^2(N_R-N_u)+8N_T(N_R-N_u)^2+9(N_R-N_u)^3]$	$O(N_T^2N_R)$
BD-LR-ZF	$2(4N_T^2\alpha+8N_T\alpha^2+9\alpha^3)+2(8N_T\alpha^2-2\alpha^2)+2[24\alpha^3-4\alpha^2+CUpdate\times(16\alpha+8)$ $+CLamda\times 4+CDelta\times 10]+2(24\alpha^3-4\alpha^2)+8N_TN_R^2-2N_R^2$	$O(N_TN_R^2)$
BD-LR-MMSE	$2(4N_T^2\alpha+8N_T\alpha^2+9\alpha^3)+2(8N_T\alpha^2-2\alpha^2)+2[24\alpha^3-4\alpha^2+CUpdate\times(16\alpha+8)$ $+CLamda\times 4+CDelta\times 10]+2(24\alpha^3-3\alpha^2+\alpha+1)+8N_TN_R^2-2N_R^2$	$O(N_TN_R^2)$

4. Simulation Results

In this Section, we compare both the computational complexities and the BER performances of the proposed algorithms with those of LC-RBD-LR-ZF algorithm in [11] and BD algorithm in [4]. In all simulation results, the channel from BS to all users are assumed to be quasi-static Rayleigh fading channel.

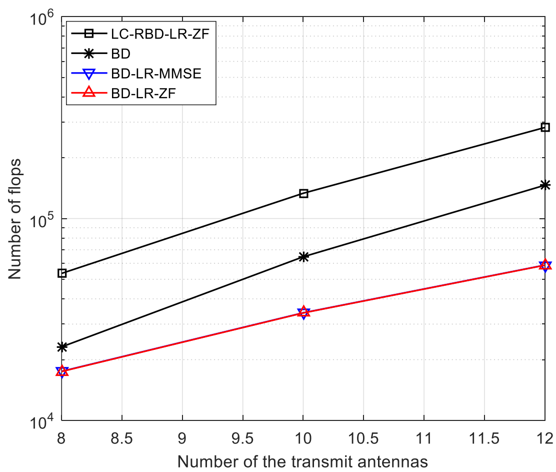

Fig. 4. Complexity comparison of all precoding algorithms

Fig. 4 demonstrates the computational complexities of LC-RBD-LR-ZF, BD, and the proposed precoders. In this scenario, N_T is varied from 8 to 12 transmit antennas. It can be seen from the figure that the complexities of the proposed precoders are significantly lower than those of the LC-RBD-LR-ZF and the BD. For example, at $N_R = N_T$ = 8 antennas, the complexity of the proposed BD-LR-MMSE is approximately equal to 32.6% and 75.5% of LC-RBD-LR-ZF and BD precoders' complexities, respectively.

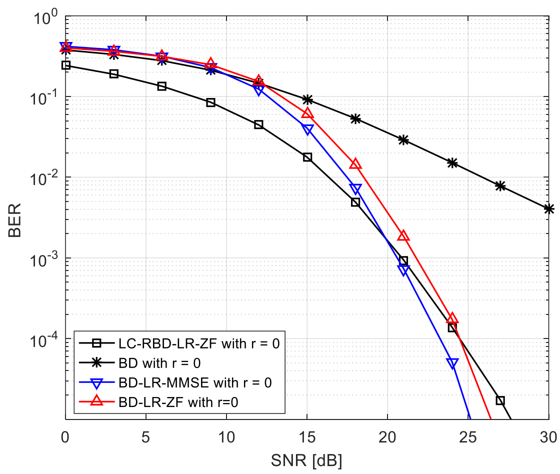

Fig. 5. The system performance with $N_T = 8$, $N_u = 2$, K = 4 in the case of uncorrelated channel

BER performances of all the precoding algorithms are illustrated in Fig. 5 to Fig. 7. In Fig. 5, the system is assumed to work in an uncorrelated MU-MIMO channel with the following parameters: $N_T = 8$, $N_u = 2$, $K = 4$, and 4-QAM modulation. In Fig. 6, we simulate the system performance under the existence of exponential correlation at both the BS side and the user side (i.e., $\mathbf{H} = \mathbf{R}_R^{1/2}\tilde{\mathbf{H}}\mathbf{R}_T^{1/2}$). The correlation coefficients are assumed to be r = 0.5 and r = 0.7. Other parameters are the same as those used to generate Fig. 5. It can be seen from both Fig. 5 and Fig. 6 that in the low and medium SNR regions, the proposed BD-LR-ZF and BD-LR-MMSE precoders underperform their LC-RBD-LR counterpart. However, at sufficiently high SNRs, they provide better system performance than LC-RBD-LR-ZF precoder. More importantly, in all scenarios, the proposed precoders outperform the BD one in the entire SNR region.

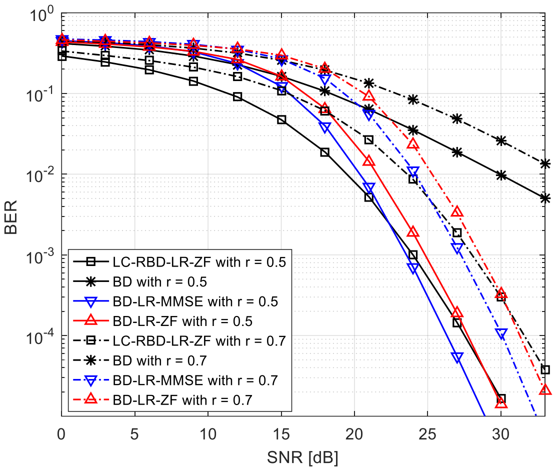

Fig. 6. The system performance with $N_T = 8$, $N_u = 2$, K = 4 in the case of correlated channel use the exponential correlation channel model, r = 0.5 and r = 0.7

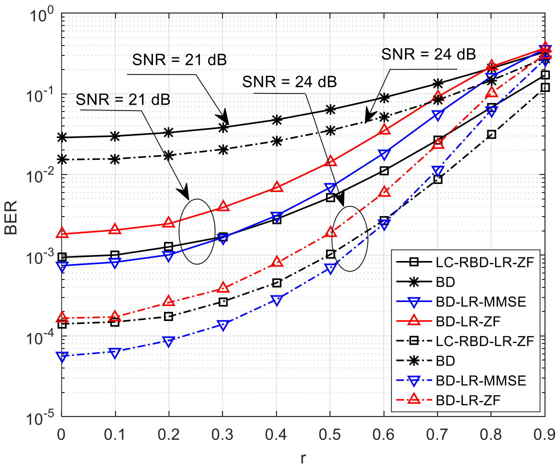

Fig. 7. The system performance according to r at SNR = 21 dB and 24 dB with $N_T = N_R = 8, K = 4, N_u = 2$

Fig. 7 illustrate the BER curves of all precoders as functions of r at SNR = 21 dB and 24 dB. Other simulation parameters are the same as those used to generate Fig. 5, i.e., $N_T = N_R = 8, K = 4, N_u = 2$, and 4-QAM modulation. We can see that for the same parameters, BD precoder performs the worst. The remaining three precoders provide nearly the same BERs, particularly when r becomes larger. Nevertheless, among the precoders, LC-RBD-LR-ZF precoder appears to be more robust as the correlation coefficient approaches unity. The simulation results in Fig. 7 also show that the correlation coefficient has an adverse effect on the system performance no matter which precoder is employed.

5. Conclusions

In this paper, we propose the BD-LR-ZF and BD-LR-MMSE precoders by combining the conventional linear precoding techniques with low-complexity ELR-SLB lattice reduction technique to improve the BER performance of MU-MIMO systems under the exponential correlation channel model. It is shown that the BD-LR-ZF and BD-LR-MMSE precoders have remarkably lower complexity than their LC-RBD-LR-ZF and BD counterpart. In addition, the BER performances of the proposed algorithms are worse than the LC-RBD-LR-ZF algorithm in the low SNR region, but better than the LC-RBD-LR-ZF algorithm in the high SNR region. BD precoder is shown to perform the worst among all the precoders. As a consequence, the proposed BD-LR-ZF and BD-LR-MMSE precoders can be potential digital beamforming techniques for practical MU-MIMO systems.

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